2018 MAθ PRECALCULUS HUSTLE ANSWERS and SOLUTIONS

1. - 44 Multiply column 1 by 2 and add to column 4 giving $\begin{vmatrix} -1 & 2 & 3 & -1 \\ 0 & 3 & 4 & -5 \\ 1 & 0 & 0 & 0 \\ 5 & 1 & -3 & 12 \end{vmatrix}$ and expand by minors on row 3. Det = $1(-1)^{3+1}\begin{vmatrix} 2 & 3 & -1 \\ 3 & 4 & -5 \\ 1 & -3 & 12 \end{vmatrix}$ = 96 – 15 + 9 + 4 – 30 – 108 = - 44.

2. 3 Factoring gives $f(x) = \frac{(x-2)(x^2+2x+4)}{(x-4)(x+5)}$. There are 2 vertical asymptotes: x=4, x=-5. Since the power of the numerator is greater, long division gives a slant asymptote of y = x + 3. So three asymptotes exist.

3. $27\sqrt{3}$ Use $A = \frac{1}{2}ab\sin C = \frac{1}{2}\cdot 9\cdot 12\cdot \sin 60^0 = 27\sqrt{3}\,un^2$

4. $\frac{13}{36}$ Let $\alpha = \sin^{-1}\left(\frac{-1}{3}\right)$ in quadrant 4, so $\cos(2\alpha) = \frac{8}{9} - \frac{1}{9} = \frac{7}{9}$. Let $\beta = \sec^{-1}\left(\frac{-13}{12}\right)$ in quadrant 2, making $\tan\beta = \frac{-5}{12}$. Then $\frac{7}{9} + \frac{-5}{12} = \frac{28-15}{36} = \frac{13}{36}$.

5. $V = \frac{\pi h^3}{16}$ Let the height be h. The diameter is $\frac{h}{2}$ and the radius would be $\frac{h}{4}$. Substituting into the volume formula, $V = \pi r^2 h = \pi\left(\frac{h}{4}\right)^2 h = \frac{\pi h^3}{16}$.

6. 4 The function has a period of 2π, a vertical shift (midline) of 1, and an amplitude of 5 which puts the minimum values at -4 so the graph crosses the x-axis twice each period. There are two periods so it crosses 4 times on the interval [-2π, 2π].

7. $y = \log_3 4$ Since $x = \log_3 y$, we know that $y = 3^x$. The equation becomes $3^y + (3^2)^y = 20$ or $(3^y)^2 + 3^y - 20 = 0$, which factors to $(3^y + 5)(3^y - 4) = 0$. The solutions are $3^y = -5$ and $3^y = 4$. Only $3^y = 4$ has a value which is $y = \log_3 4$.

8. B When simplified, B becomes $\csc^2 x + \cot^2 x = 1$ but the Pythagorean Trig Identity has a minus sign. The other statements are true using co-functions in A and Pythagorean Identities in C and D.

9. -5, -1, 4 p(-x) has a solution of 1 so the polynomial has a solution of -1. Synthetically dividing gives a quotient of $x^2 + x - 20$. That factors into (x + 5)(x – 4) = 0 and results in 3 solutions: -5, -1, and 4.

10. $6\sqrt{3}$ The hexagon can be broken into 6 equilateral triangles with side length of 2, the integral solution of the equation. The area then becomes $A = 6\left(\frac{1}{2}\cdot 2\cdot 2\sin 60^0\right) = 6\sqrt{3}$.

11. 22 Powers of 8 repeat the digits {8, 4, 2, 6}. Powers of 3 repeat the digits {3, 9, 7, 1}. Powers of 7 repeat the digits {7, 9, 3, 1}. 2018 divided by 4 has remainder of 2, so we need to use the second value in the sequences. 4 + 9 + 9 = 22.

12. - 2 Finding the determinant of each side gives -11x – 14 = 20 + 6x so x = - 2.

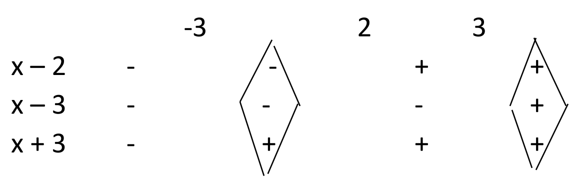
13. - 6 $f \circ f(x) = (x^2 + 3x)^2 + 3(x^2 + 3x) = x^4 + 6x^3 + 9x^2 + 3x^2 + 9x$
$= x^4 + 6x^3 + 12x^2 + 9x = x(x^3 + 6x^2 + 12x + 9)$
The sum of solutions for the cubic is –B/A, or -6/1 plus the value of x=0. Sum = -6.

14. $(4\sqrt{3}, -4)$ The original vector has a magnitude of 8 and a direction angle of -60^0 + 180^0 = 120^0. Rotating 150^0 clockwise puts the angle at -30^0 with the magnitude remaining at 8. The new vector's terminal point is $(8\cos(-30^0), 8\sin(-30^0))$ = $\left(4\sqrt{3}, -4\right)$.

15. $\left(\frac{2}{3}, 1\right) \cup (1, 2) \cup (3, \infty)$

The argument of the log must be positive.

For $\frac{x-2}{(x-3)(x+3)} > 0$, we get

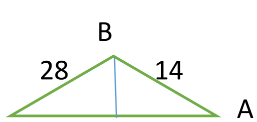


The base of the log must be positive but not equal to 1. 3x – 2 > 0 gives x > 2/3.
3x – 2 $\neq 1$ gives $x \neq 1$. Combining these domains results in $\left(\frac{2}{3}, 1\right) \cup (1, 2) \cup (3, \infty)$.

16. $-32,\ or - 32 + 0i$ $xy = 4 \cdot 8\ cis\left(\frac{3\pi}{4} + \frac{\pi}{4}\right) = 32cis\pi = 32cos\pi + 32isin\pi = 32(-1) + 0.$

17. $\frac{73}{81}$ $\sin^4x + \cos^4x = \sin^4x + 2\sin^2x\cos^2x + \cos^4x - 2\sin^2x\cos^2x = (\sin^2x + \cos^2x)^2 - (1/2)(4\sin^2x\cos^2x)$
$= 1 - (1/2)(\sin 2x)^2 = 1 - (1/2)(4/9)^2 = 1 - (8/81) = 73/81$

18. 1



Since we are given SSA information and the opposite side is smaller than the adjacent side, we need to check the height. h = 28sin30^o = 14 which equals the opposite side and results in one right triangle.

19. 0 $\sum_{i=1}^{12}(\cos(i\pi) + \sin(i\pi)) = \sum_{i=1}^{12}\cos(i\pi) + \sum_{i=1}^{12}\sin(i\pi) = \sum_{i=1}^{12}\cos(i\pi) + 0$

= $\sum_{i=1}^{12}\cos(i\pi)$ = 0 since cosine of odd multiples of π = -1 and cosine of even multiples of π = 1. The sine of all multiples of π = 0.

20. $\frac{4}{3}$ If a, b, c, and d are the solutions of the equation, then the sum of the reciprocals, $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}$, is $\frac{abc+acd+abd+bcd}{abcd}$. The product of the roots of the equation is $\frac{6}{3}$ and the sum of the roots taken 3 at a time is $-\left(\frac{-8}{3}\right)$. Dividing, $\frac{8}{3} \div \frac{6}{3} = \frac{4}{3}$.

21. $\frac{1}{3}$ $P = \pi \div \left(\frac{3\pi}{4}\right) = \frac{4}{3}$ *and* $A = 4$. *Thus,* $\frac{P}{A} = \frac{4}{3} \div 4 = \frac{1}{3}$.

22. ½ $\log_2 36 = \frac{\log 36}{\log 2}$ *and* $\log_3 36 = \frac{\log 36}{\log 3}$ so the reciprocals, when added, give $\frac{\log 2}{\log 36} + \frac{\log 3}{\log 36} = \frac{\log 6}{\log 36} = \log_{36} 6 = 1/2$.

23. $\frac{340}{3}$ *or* $113\frac{1}{3}$ The "drop" distances form the sequence: 20, 14, 9.8, …

The "bounce" distances form the sequence: 14, 9.8, 6.86, …

Both are infinite geometric sequences, so using the formula $= \frac{a_1}{1-r}$,

we get $S = \frac{20}{1-.7} + \frac{14}{1-.7} = \frac{34}{.3} = \frac{340}{3}$ feet.

24. $\frac{9}{8}$ Cross-multiplying gives the equation $r - \frac{1}{3}r\, sin\theta = 3$ *or* $3r = r sin\theta + 9$

Substituting rectangular values gives $3\sqrt{x^2 + y^2} = y + 9$

Squaring both sides results in $9x^2 + 9y^2 = y^2 + 18y + 81$ which is an ellipse.

Grouping and completing the squares: $9x^2 + 8y^2 - 18y = 81$

$9x^2 + 8\left(y^2 - \frac{9}{4}y + \frac{81}{64}\right) = 81 + \frac{81}{8}$

$9x^2 + 8\left(y - \frac{9}{8}\right)^2 = \frac{729}{8}$ with center $(0, \frac{9}{8})$

25. 10 $\prod_{k=1}^{9}\left(1 + \frac{1}{k}\right) = \sum_{k=1}^{9}\left(\frac{k+1}{k}\right) = \frac{2}{1}\cdot\frac{3}{2}\cdot\frac{4}{3}\cdot \cdots \cdot\frac{9}{8}\cdot\frac{10}{9} = 10.$